**Instructions**: Try these problems from the textbook as practice (answers below).

## Problems

3-10. Determine whether or not $\mathbf{F}$ is a conservative vector field. If it is, find a function $f$ such that $\mathbf{F} = \nabla f$.

5. $\mathbf{F}(x,y) = y^2 e^{xy}\mathbf{i} + (1+xy)e^{xy}\mathbf{j}$
7. $\mathbf{F}(x,y) = (ye^x + \sin(y))\mathbf{i} + (e^x + x\cos(y))\mathbf{j}$
9. $\mathbf{F}(x,y) = (y^2\cos(x) + \cos(y))\mathbf{i} + (2y\sin(x) - x\sin(y))\mathbf{j}$

15. Show that the vector field $\mathbf{F} = \langle ye^{xy}, xe^{xy}\rangle$ is conservative and find a potential function $f$. Evaluate $\displaystyle\int_C \mathbf{F}\cdot d\mathbf{r}$ where $C$ is given by $x = \sin(t\pi/2)$, $y = e^{t-1}$ and $0 \le t \le 1$. Evaluate $\displaystyle\int_C \mathbf{F}\cdot d\mathbf{r}$ by first replacing $C$ with a line segment that has the same endpoints.

17-24. Find a function $f$ such that $\mathbf{F} = \nabla f$ and use it to evaluate $\displaystyle\int_C \mathbf{F}\cdot d\mathbf{r}$ along the given curve $C$.

19. $\mathbf{F}(x,y) = x^2y^3\mathbf{i} + x^3y^2\mathbf{j},\ C : \mathbf{r}(t) = \langle t^3 - 2t, t^3 + 2t\rangle,\ 0 \le t \le 1$
21. $\mathbf{F}(x,y,z) = 2xy\mathbf{i} + (x^2 + 2yz)\mathbf{j} + y^2\mathbf{k}$, $C$ is the line segment from $(2,-3,1)$ to $(-5,1,2)$
23. $\mathbf{F}(x,y,z) = yze^{xz}\mathbf{i} + e^{xz}\mathbf{j} + xye^{xz}\mathbf{k},\ C : \mathbf{r}(t) = \langle t^2+1, t^2-1, t^2-2t\rangle, 0 \le t \le 2$

25. Show that $\displaystyle\int_C 2xe^{-y}\,dx + (2y - x^2e^{-y})\,dy$ where $C$ is any path from $(1,0)$ to $(2,1)$ is independent of path and evaluate the integral.

29. Find the work done by the force field $\mathbf{F}(x,y) = x^3\mathbf{i} + y^3\mathbf{j}$ in moving an object from $P(1,0)$ to $Q(2,2)$.

## Answers

5. $f(x,y) = ye^{xy} + K$
7. $f(x,y) = ye^{x} + x\sin y + K$
9. $f(x,y) = y^2\sin x + x\cos y + K$
15. $f(x,y) = e^{xy} + K$, $e^2 - 1$
19. $f(x,y) = \dfrac{1}{3}x^3y^3$, $-9$
21. $f(x,y,z) = x^2y + y^2z$, $30$
23. $f(x,y,z) = ye^{xz}$, $4$
25. $\dfrac{4}{e}$
29. $\dfrac{31}{4}$